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One of the most experimentally testable explanations for the origin of the baryon asymmetry of the universe is that it was created during the electroweak phase transition, in the minimal supersymmetric standard model. Previous efforts have focused on the current for the difference of the two Higgsino fields, $H_1 - H_2$, as the source of biasing sphalerons to create the baryon asymmetry. We point out that the current for the orthogonal linear combination, $H_1 + H_2$, is larger by several orders of magnitude, making it possible to have naturally small CP-violating phases, as required by electric dipole moment constraints.

1. A highly constrained proposal for the origin of baryonic matter in the universe is electroweak baryogenesis in the minimal supersymmetric standard model (MSSM) [1–5]. Unlike many other baryogenesis mechanisms, this one has strong prospects for being falsified in upcoming experiments, due to its need for some light exotic particles, notably a right-handed top squark which is lighter than the top quark [6,7].

The basic mechanism [8] is intuitively clear: particles interact in a CP-violating manner with bubble walls, which form during the first order electroweak phase transition, when the temperature of the universe was near $T = 100$ GeV. This causes a buildup of a left-handed quark density in excess of that of the corresponding antiquarks, and an equal and opposite right-handed asymmetry, so that there is initially no net baryon number. The left-handed quark asymmetry biases anomalous sphaleron interactions, present within the standard model, to violate baryon number preferentially to create a net quark density. The resulting baryon asymmetry of the universe (BAU) soon falls inside the interiors of the expanding bubbles, where the sphaleron interactions are shut off, and thus baryon number is safe from subsequent sphaleron-induced relaxation to zero.

Despite the simplicity of this idea, a quantitatively accurate calculation is difficult to carry out. Various approximations have been made, leading to a variety of formalisms which give somewhat conflicting predictions for the dependence of the BAU on the parameters of the MSSM. Although most authors agree on the diffusion equations which govern the generation of the left-handed quark asymmetry, there is less consensus about how to derive the source terms which appear in these equations.

In a previous paper [4], we advocated an approach based on the classical force on particles [9,10] due to their spatially varying masses as inside the bubble wall. It is straightforward to solve for this force, put it into the Boltzmann equations, and derive corresponding diffusion equations. No *ad hoc* assumptions are needed; one only requires that the width of the wall be significantly larger than thermal de Broglie wavelengths of particles in the plasma, to justify an expansion in derivatives of the background Higgs field, which constitutes the bubble wall. Detailed calculations of the wall width confirm that this is indeed a good approximation [11].

One puzzling conflict between our earlier work [4] and that of others was that we derived a source which remains large when the ratio of the two Higgs fields, $H_2(x)/H_1(x)$, is constant inside the bubble wall. Other authors [1–3] found sources proportional to the derivative of this quantity. Careful analyses of the shape of the wall have shown that in fact H_2/H_1 remains constant to within a part in $10^2 - 10^3$ [12,7], so that the dependence on $d(H_2/H_1)/dx$, if correct, would result in a large suppression of the generated BAU.

In this Letter we explain the origin of the apparent discrepancy: it is the result of different choices about which linear combination of Higgsino currents is used to source the diffusion equations. References [1–3] considered only the combination $H_1 - H_2$, based on the observation [1] that the orthogonal source, $H_1 + H_2$, is driven to zero in the limit that interactions proportional to the top quark Yukawa couplings are infinitely fast. Although this approximation is convenient because it simplifies the network of coupled diffusion equations, in the present application it can lead to a serious underestimate of the BAU, which would have to be compensated by assuming that the underlying CP phases of the MSSM are of order unity. Although it may be possible to fine-tune various phases to cancel each others' contributions to highly constrained neutron, electron and atomic electric dipole moments, it is reassuring to see that such fine tuning is not after all required: a large enough BAU can be produced without resorting to large phases. We wish to demonstrate how this comes about, while at the same time updating our earlier computation so as to give a quantitatively accurate determination of the BAU as a function of relevant parameters in the MSSM.

2. To explain the how the classical force mechanism works, we first review the simple case of a top quark with a spatially varying complex mass [9], $m(x) = yH(x)e^{i\theta(x)}$. By solving the Dirac equation $(i\not\partial - |m|\cos\theta - i|m|\sin\theta\gamma_5)\psi$ to first order in derivatives (the WKB approximation), one finds that a particle of energy E experiences a spin (s) dependent force

$$F = \frac{dp}{dt} = -\frac{(m^2)'}{2E} + \frac{s}{2E^2}\left(m^2\theta'\right)'. \qquad (1)$$

The spin dependent part of the force has opposite sign for antiparticles, which causes the distributions of like-helicity fermions and antifermions to be separated in the vicinity of the wall. For relativistic particles, we can approximate helicity by chirality, and speak of spatially varying chemical potentials, $\mu_{L,R}$ for the left- and right-handed components; these are related to the asymmetry between the particle and antiparticle densities by $n(x) - \bar{n}(x) = \mu(x)T^2/6$. Diffusion equations can be derived by inserting the force (1) into the Boltzmann equation, doing an expansion in moments of the distribution functions, and truncating the expansion. They have the form

$$-D\mu_{L}{}''_{i} - v_w\mu_{L}{}'_{i} + \Gamma\mu_{L\,i} = S_i(x), \qquad (2)$$

where D is the diffusion coefficient (of order the inverse mean free path), v_w is the bubble wall velocity, and Γ is a damping rate representing decays or inelastic collisions of the left-handed fermions. The source term for each species is related to the classical force through a thermal average [13]

$$S(x) = -\frac{v_w D}{\langle \vec{v}^2\rangle}\langle v_x F(x)\rangle'. \qquad (3)$$

Once the left-handed quark density is found from eq. (2), the density of baryons generated by sphalerons can be computed by integrating $\mu_L(x) \equiv \sum_{q_i}\mu_{L\,i}$ in front of the wall:

$$n_B = \frac{9\Gamma_{\rm sph}T^2}{2v_w}\int_0^\infty \mu_L(x)\, e^{-c_b\Gamma_{\rm sph}x/v_w}\, dx, \qquad (4)$$

where $\Gamma_{\rm sph}$ is the diffusion rate of the Chern-Simons number as measured by lattice simulations [14]: $\Gamma_{\rm sph} = (20\pm 2)\alpha_w^5 T$. The exponential accounts for sphaleron-induced relaxation of the baryon asymmetry back to zero due to restoration of thermal equilibrium, in the limit of a very slowly moving wall. The coefficient c_b depends on the squark spectrum: it equals to 45/4 if all squarks are light and 72/7 if only the right handed stop is light.

3. When the above picture is adapted to the MSSM some complications occur, because the quarks do not get complex masses and hence are not directly sourced. The chargino mass matrix does contain complex phases however, and the chiral asymmetry which develops in the chargino sector induces one for the quarks because of the strong coupling between Higgsinos and the top quark. This system involves not just one, but many coupled diffusion equations. We will show that they can, nevertheless, be reduced to a single equation by reasonable approximations.

We start the treatment of the MSSM by deriving the source term analogous to $S(x)$ in eqs. (2–3). The mass term in the Lagrangian for the charginos is

$$\overline{\psi}_R M_\chi \psi_L = (\overline{\widetilde{w}^+},\ \overline{\widetilde{h}_2^+})_R \begin{pmatrix} m_2 & gH_2 \\ gH_1 & \mu \end{pmatrix} \begin{pmatrix} \widetilde{w}^+ \\ \widetilde{h}_1^+ \end{pmatrix}_L \qquad (5)$$

plus the Hermitian conjugate, $\overline{\psi}_L M_\chi^\dagger \psi_R$. Because there are two Higgs doublets, H_1 and H_2, there are two corresponding Higgsino fields $\widetilde{h}^+_{1,2}$. $\widetilde{w}^+$ is the wino, superpartner of the W boson, and g is the weak gauge coupling. The complex phases of the wino mass m_2 and the μ parameter are the origin of a CP violating force. By again solving the Dirac equation in the WKB approximation, one can find the spin-dependent forces which act on the two mass eigenstates, with masses $m^2_\pm$. Since Higgsinos couple strongly to top quarks, we are interested in the one which smoothly evolves into the Higgsino state in front of the wall, where $H_{1,2}\to 0$. The spin-dependent part of the force has the same form as in eq. (1), with the spatially varying phase given by

$$m^2_\pm\theta'_h = \mp\frac{g^2{\rm Im}(m_2\mu)}{(m^2_+ - m^2_-)}\left(H_1H_2' + H_1'H_2\right), \qquad (6)$$

where the Higgsino-like mass eigenvalue, $m^2_{\tilde h}(x)$, is m^2_- (the lighter one) if $|\mu|^2 < |m_2|^2$, and m^2_+ otherwise. We remark that the effective WKB expansion parameter θ'_h/E remains small even when mass gap $m^2_+ - m^2_-$ attains its minimum value; this corresponds to $|m_2|\simeq|\mu|$, where one has parametrically $\theta'_h/E \simeq gH_i/wE|\mu| \ll 1$, because for typical wall widths $wE \gtrsim 20$ [11].

Having specified the source term, we must now deal with the diffusion equations in which it appears. In general, these are a set of coupled equations for the two Higgsinos ($\tilde h_{1,2}$), the winos, 6 flavors of right-handed quarks, 3 generations of left-handed quark doublets, and all the superpartners which are light enough to be present at the temperature T. Following Huet and Nelson [1], we will make the simplifying assumption that the supergauge interactions (*e.g.,* the coupling of winos to quarks and squarks) are in thermal equilibrium, so that particle and corresponding sparticle chemical potentials are equal to each other, species by species; in particular gaugino chemical potentials are driven to zero. Also the strong sphaleron interactions, enforcing equality of total left and right chiral quark asymmetries, are fast in comparision with the relevant diffusion time scale $t_D \simeq D/v_w^2 \gtrsim 10^3$, and can be taken to be in equilibrium.

With these simplifications, the diffusion equation network can be reduced to three, for the potentials of the

Higgsinos $\tilde{h}_1$, $\tilde{h}_2$ and, say, the left-handed third generation quark doublet q_3. The most important interactions coupling these species come from the potential

$$V = y\bar{q}_3 H_2 t_R + yA\tilde{q}_3 H_2 \tilde{t}_R^c + y\mu\tilde{q}_3^* H_1 \tilde{t}_R + \mu\tilde{h}_1\tilde{h}_2 \quad (7)$$

plus hermitian conjugate terms. The rates associated with the trilinear interactions can be parametrized as Γ_y, $\Gamma_{y\mu}$, Γ_{yA} in a self-evident notation. In addition there are helicity flipping interactions with rate Γ_{hf} coupling $\tilde{h}_1$ and $\tilde{h}_2$, due to the μ term, and in the broken phase inside the bubble ($x < 0$), the top quark Yukawa coupling becomes the top mass, which causes helicity flips between the left- and right-handed top quarks, q_3 and t_R, with a rate of Γ_m. Defining the diffusion operator $\mathcal{D}_i = -6(D_i\partial_x^2 + v_w\partial_x)$, the equations are

$$\begin{aligned}
&\mathcal{D}_{\tilde{h}}\mu_{\tilde{h}_1} + \Gamma_{y\mu}(\mu_{\tilde{h}_1} - c_3\mu_{q_3}) + \Gamma_{hf}(\mu_{\tilde{h}_1} + \mu_{\tilde{h}_2}) = S \\
&\mathcal{D}_{\tilde{h}}\mu_{\tilde{h}_2} + (\Gamma_y + \Gamma_{yA})(\mu_{\tilde{h}_2} + c_3\mu_{q_3}) + \Gamma_{hf}(\mu_{\tilde{h}_1} + \mu_{\tilde{h}_2}) = S \\
&\mathcal{D}_q\mu_{q_3} + \tilde{c}\left[-(\Gamma_y + \Gamma_{yA})(\mu_{\tilde{h}_2} + c_3\mu_{q_3})\right. \\
&\qquad \left. + \Gamma_{y\mu}(\mu_{\tilde{h}_1} - c_3\mu_{q_3}) + c_3\,\Gamma_m\mu_{q_3}\,\theta(-x)\right] = 0. \quad (8)
\end{aligned}$$

The coefficients $\tilde{c}$ and c_3 are numbers of order unity, depending on the ratio of squark masses to temperature. If all squarks are light, $c_3 = 7/2$, and if only $\tilde{t}_R$ is light, as needed to get a strongly first order phase transition, $c_3 = 23/13$. Because of the hierarchy $D_q \ll D_h$ we can approximately solve the third equation by taking its interactions to be in thermal equilibrium, *i.e.,* ignoring the term $\mathcal{D}_q\mu_{q_3}$, which enables us to eliminate μ_{q_3}. (Hence we do not need to evaluate $\tilde{c}$ at this level of approximation.) It is useful to form the linear combinations

$$\mu_\pm = \frac{1}{2}(\mu_{\tilde{h}_1} \pm \mu_{\tilde{h}_2}) \quad (9)$$

and also define $\Gamma_\pm = \frac{1}{2}\left(\Gamma_{y\mu} \pm (\Gamma_y + \Gamma_{yA})\right)$. The remaining two equations then take the form

$$\begin{aligned}
&\mathcal{D}_{\tilde{h}}\mu_+ + \left(2\Gamma_{hf} + \Gamma_+ - \Gamma_-^2\left(\frac{\theta(-x)}{\Gamma_+ + \frac{1}{2}\Gamma_m} + \frac{\theta(x)}{\Gamma_+}\right)\right)\mu_+ \\
&\qquad + \theta(-x)\frac{\Gamma_-\Gamma_m}{2\Gamma_+ + \Gamma_m}\mu_- = S \quad (10) \\
&\mathcal{D}_{\tilde{h}}\mu_- + \theta(-x)\frac{\Gamma_m}{2\Gamma_+ + \Gamma_m}\left(\Gamma_+\mu_- + \Gamma_-\mu_+\right) = 0. \quad (11)
\end{aligned}$$

It is noteworthy that only the linear combination μ_+ gets directly sourced in our WKB treatment, whereas the source for the combination μ_- vanishes (in this respect ref. [4] was in error). This is in contrast to papers which treat the particle reflections from the wall quantum mechanically; these works find that both μ_+ and μ_- are sourced. Here we point out that, within the quantum reflection formalisms, it is still true that μ_+ has a larger source than μ_-, because $H_1'H_2 + H_2'H_1$ is larger than $H_1'H_2 - H_2'H_1$ [15]. However nobody has heretofore considered the former because of the unfortunate approximation of imposing equilibrium of the Yukawa interactions, which forces μ_+ to zero.

We can reduce the remaining two diffusion equations to a single one by treating the ratio Γ_-/Γ_+ perturbatively. At zeroth order in Γ_-/Γ_+ the equations decouple, and $\mu_\pm(x)$ can be found using standard Green's function methods [16]. The equilibrium condition associated with eq. (8) determines μ_{q_3} in terms of $\mu_\pm$, and strong sphaleron equilibrium implies that the corresponding first two generation potentials are $\mu_{q_1,q_2} = c_{1,2}\mu_{q_3}$, with $c_{1,2} = -1/2$ if all squarks are are light compared to T, and $c_{1,2} = -4/13$ if all but $\tilde{t}_R$ are heavy. The total left-handed potential $\mu_L = \sum_i \mu_{q_i}$, must be numerically integrated in eq. (4) to obtain the baryon asymmetry.

4. We have carried out the above procedure to study how the BAU depends on the velocity of the bubble wall v_w, the wall width w (appearing in the Higgs field profile as $H_0(1-\tanh(x/w))/2$), the diffusion constant $D_{\tilde{h}}$, the interaction rates $\Gamma_\pm$, Γ_m, Γ_{hf}, $\tan\beta = \langle H_2\rangle/\langle H_1\rangle$ (at zero temperature) and the chargino mass parameters μ and m_2. We take as our preferred values $w = 10/T$ [11], $D_{\tilde{h}} = 20/T$, $\Gamma_+ = 0.02T$, $\Gamma_- = 0.25\Gamma_+$, $\Gamma_{hf} = 0.013T$, $\Gamma_m = 0.007T$, $\tan\beta = 3$, $\mu = 120$ GeV, $m_2 = 115$ GeV, and $v_w = 0.1$. In the following figures where certain of these quantities are varied, those which are not specified have the above values. Notice that our results scale linearly with Γ_-/Γ_+ in the regime where it is small. It is customary to express the BAU as a scaled ratio of baryons to photons, $\eta_{10} = 10^{10}n_B/n_\gamma = 7 \times 10^{10}n_B/s$, where $s = (2\pi^2/45)g_*T^3$, and we take the number of degrees of freedom in the MSSM to be $g_* = 110$. Current limits from big bang nucleosynthesis give $3 \lesssim \eta_{10} \lesssim 4$. Finally, we have accounted for the Boltzman suppression when charginos become heavy by multiplying the source S by a suppression factor $n(m/T)/n(0) = \frac{1}{2}(m/T)^2K_2(m/T)$, where K_2 is the modified Bessel function.

In figure 1, the BAU is plotted as a function of wall velocity, varying the Higgsino diffusion coefficient to obtain the different curves. We have taken the CP-violating phase appearing in $\mathrm{Im}(m_2\tilde{\mu})$ (eq. (6)) to be maximal; to satisfy $\eta_{10} \cong 3$, one should rescale this phase accordingly. The efficiency of baryogenesis tends to peak for $v_w = 0.01 - 0.03$. Interestingly, recent work on bubble expansion has suggested just such a range of small values in the MSSM [17]. Similar looking curves can be obtained by varying other parameters. The maximum values of the BAU (occurring near $v_w = 1.5 \times 10^{-2}$) are given in Table 1.

parameter	$\frac{6}{T} < w < \frac{20}{T}$	$4 < 10^3\frac{\Gamma_m}{T} < 13$	$20 < D_{\tilde{h}}T < 100$
η_{10}	$6000 - 2500$	$2000 - 3300$	$3000 - 5900$

Table 1: Dependence of maximum BAU on other parameters.

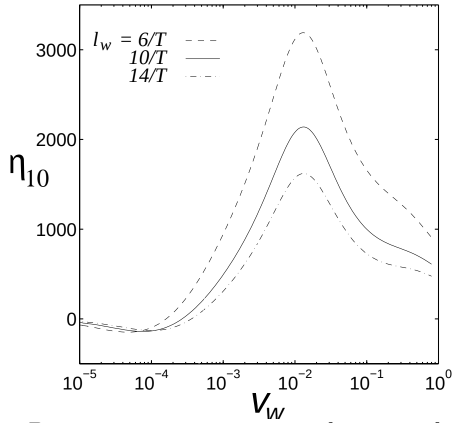

Figure 1: Baryon asymmetry η_{10} as a function of wall velocity v_w for wall width $w = 6/T$, $10/T$ and $14/T$.

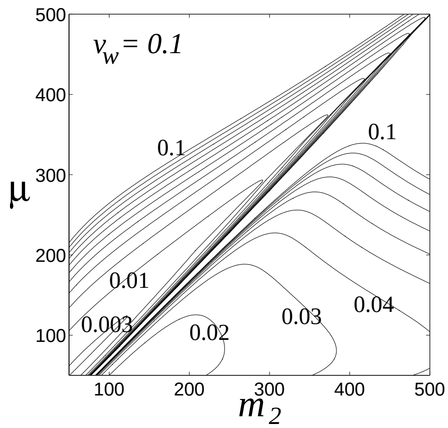

Figure 2: Contours of constant CP-violating phase, $\delta = \arg(m_2\mu)$, giving $\eta_{10} = 3$, in the plane of the chargino mass parameters μ and m_2 (in GeV units).

We can also consider the dependence of the BAU on the chargino mass parameters, μ and m_2. Figure 2 exhibits contours of constant magnitude of CP-violating phase, $\delta = \arg(m_2\mu)$, giving the desired BAU, $\eta_{10} = 3$. The steepness of contours near $m_2 = \mu$ is caused by the fact that the requisite δ changes sign along this line. The figure shows that CP phases as small as $\delta =$ a few $\times 10^{-3}$ can suffice for baryogenesis in the MSSM.

5. In summary, we have presented a quantitatively accurate analysis of the baryon asymmetry of the universe in the MSSM, using the classical force mechanism, which allows a consistent computation of the sources appearing in the diffusion equations. A CP-violating force acts on Higgsinos while they cross the bubble walls causing a particle-antiparticle separation in charginos, which then gets partially transformed to a chiral quark asymmetry that biases sphalerons to produce baryons. Unlike previous authors, we have not assumed that the two components of the Higgsino, $\tilde{h}_1$ and $\tilde{h}_2$, reach complete chemical equilibrium through Yukawa and helicity-flipping interactions, and we showed that a large enhancement of the BAU can result. Further details of our formalism and calculations will be presented in an upcoming paper.

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[15] Ref. [5], working in the thin wall limit, notes that μ_- does get a source at next order ($O(H^4/T^4)$) in an expansion in powers of the background Higgs fields.

[16] $\mu_+(x) = \int_{-\infty}^{\infty} G_+(x-x')S(x')dx'$, with $G_+(x) = [D_{\tilde{h}}(k_+ - k_-)]^{-1}[\theta(x)e^{-k_+ x} + \theta(-x)e^{-k_- x}]$, $k_\pm = (v_w/2D_{\tilde{h}})(1 \pm [1 + 2\bar{\Gamma} D_{\tilde{h}}/3v_w^2]^{1/2})$, and $\bar{\Gamma} = 2\Gamma_{hf} + \Gamma_+$. Once μ_+ is known, μ_- is determined via eq. (11) to first order in Γ_-/Γ_+ in a similar way: $\mu_-(x) = \Gamma_m \Gamma_-(2\Gamma_+ + \Gamma_m)^{-1} \int_{-\infty}^{0} G_-(x,x')\mu_+(x')dx'$, with $G_-(x,x') = -[\gamma\alpha D_{\tilde{h}}]^{-1} e^{-v_w x'/D_{\tilde{h}} - \alpha x}$ in the region where it is needed ($x > 0$), $\gamma = 2 + \Gamma_m/\Gamma_+$ and $\alpha = -(v_w/2D_{\tilde{h}})(1 + [1 + 2\Gamma_m D_{\tilde{h}}/3\gamma v_w^2]^{1/2})$.

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